

所別	科目	准考證號碼 (請考生填入)	考試日期	節次	第 1 頁/ 共 1 頁
電機工程研究所	工程數學		94年5月1日	第一節	

一、  $x^2 y'' - xy' + 2y = 0$  ,  $y(1) = 2$  ,  $y'(1) = -4$  ; Solve this differential equation. (10%)

二、  $y' = \frac{y^2}{x} + \frac{2y}{x} - \frac{3}{x}$  ,  $y(1) = 4$  ; Solve this differential equation. (10%)

三、  $f(t) = 3t^2 + \int_0^t f(t-\tau) e^{-\tau} d\tau$  , Use the Laplace transform to solve  $f(t) = ?$  (10%)

四、 Find the inverse Laplace transform  $\mathcal{L}^{-1} \left[ \frac{3s+2}{s^2-8s+8} \right] = ?$  (10%)

五、 One corner of a rectangular parallelepiped is at  $(2, -1, 2)$  and three incident sides extend from this point to  $(1, 0, 1)$  ,  $(6, -4, 8)$  , and  $(-2, -3, 4)$ . Find the volume of this solid ? (10%)

六、 Answer the following questions briefly.

(1) State the definition of Hermitian matrices and give an illustrative example. (5%)

(2) State the definition of unitary matrices and give an equivalent statement in terms of row vectors. (5%)

七、 Let  $A = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$ . (1) Find Rank  $(A)$ . (5%) (2) Find all solutions of  $Ax = 0$ . (5%)

八、 Let  $A = \begin{bmatrix} 3 & -1 & 2 \\ 2 & 0 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ . Answer the following questions.

(1) Find a matrix  $P$  and a diagonal matrix  $D$  so that  $P^{-1}AP = D$ . (5%)

(2) Compute  $A^{100}$ . (5%)

九、 Let  $A = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{bmatrix}$ . Find the determinant of  $A$ . (10%)

十、 Let  $f$  be the function defined as  $f(x) = u(x+2) - u(x-2)$ ,  $\forall x \in \mathcal{R}$ , where  $u(\cdot)$  denotes the unit step function.

(1) Find the Fourier transform of  $f(x)$ . (5%)

(2) Use the formula of the inverse Fourier transform and the answer of (1) to find the Fourier transform of  $g(x) = \sin(2x)/x$ . (5%)