

科目	考試日期	節次	准考證號碼
微積分	92年5月11日	第1節	

一、選擇題 (共 20 題；每題 3 分)

- 若拋物線 $y = x^2 + ax + b$ 與曲線 $y = x^3$ 在點 $(1, 1)$ 相切，則 $(a, b) =$
 - $(1, -1)$
 - $(-1, 1)$
 - $(-2, 2)$
 - $(2, -2)$
 - 以上皆非。
- 函數 $f(x) = x^2 - xe^x + e^x$ 之圖形與 x 軸相交於幾點？
 - 0
 - 1
 - 2
 - 3
 - 4
- Use the trapezoidal rule with 4 subintervals to estimate $\int_1^3 \frac{1}{x} dx$, the answer will be
 - $\frac{67}{60}$
 - $\frac{67}{40}$
 - $\frac{67}{50}$
 - $\frac{67}{30}$
 - $\frac{67}{20}$
- $\lim_{x \rightarrow 3} \frac{\sin(x-3)}{x^2 - x - 6} =$
 - 0
 - $\frac{1}{5}$
 - $\frac{1}{2}$
 - 1
 - 不存在。
- 下列何者為真？
 - 若 $\lim_{t \rightarrow \infty} \int_t^t f(x) dx$ 存在，則 $\int_{-\infty}^{\infty} f(x) dx$ 必存在
 - 若函數 f 在 (a, b) 連續，則 f 在 (a, b) 可微分
 - 若函數 f 在 $[a, b]$ 連續，則 f 在 $[a, b]$ 可積分
 - 若函數 f 在 $[a, b]$ 不連續，則 f 在 $[a, b]$ 不可積
 - 若 $\lim_{x \rightarrow \infty} (f(x) + g(x))$ 存在，則 $\lim_{x \rightarrow \infty} f(x)$ 和 $\lim_{x \rightarrow \infty} g(x)$ 必存在。

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6. 設 $f(x) = \int_{\pi/2}^x t \sin t dt$, $\frac{\pi}{2} \leq x \leq 4$, 則 $f(x)$ 之極大值為

- (A) 1
 (B) π
 (C) $4\sin 4$
 (D) $\frac{\pi}{2}$
 (E) $\pi - 1$

7. If $A = \{(x, y) \mid y \geq \sin x, y \leq 1 + \cos x, 0 \leq x \leq \frac{\pi}{2}\}$, the area of A is

- (A) $\pi + 1$
 (B) π
 (C) $\pi + 1/2$
 (D) $\frac{\pi}{2}$
 (E) $\frac{\pi + 1}{2}$

8. 下列敘述何者為錯誤的？

- (A) $\int_0^{2\pi} |\sin x| dx = 4 \int_0^{\pi/2} \sin x dx$
 (B) $\int_{-1}^1 \frac{1}{x} dx = 0$
 (C) $\int_0^1 \frac{1}{1+x^2} dx = \frac{\pi}{4}$
 (D) $\int_0^{\infty} \frac{dx}{(x+2)^2} = 1$
 (E) 以上皆非。

9. $\sum_{n=1}^{+\infty} a_n$ is a given series, which of the following statement is not true?

- (A) If $\lim_{n \rightarrow +\infty} \sum_{k=1}^n a_k$ exists, $\sum_{n=1}^{+\infty} a_n$ converges.
 (B) If $\lim_{n \rightarrow +\infty} a_n \neq 0$, $\sum_{n=1}^{+\infty} a_n$ diverges.
 (C) If $\sum_{n=1}^{+\infty} |a_n|$ converges, $\sum_{n=1}^{+\infty} a_n$ converges.
 (D) If $\sum_{n=1}^{+\infty} a_n$ diverges, $\sum_{n=1}^{+\infty} |a_n|$ diverges.
 (E) If $\sum_{n=1}^{+\infty} a_n$ converges, $\sum_{n=1}^{+\infty} (-1)^n a_n$ converges.

10. If A, B, C are matrices, which of the following is true?

- (A) If AB exists, $AB=BA$.
 (B) If A is a skew symmetric matrix, the diagonal elements of A are all zero.

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(C) $\text{trace} \begin{pmatrix} 1 & 7 & 3 \\ -1 & 5 & 2 \\ 2 & 3 & -4 \end{pmatrix} = -20.$

(D) If A is a $n \times n$ matrix, c is a real number, $\det(cA) = c \det(A).$

(E) If $\det(A) = 0$, the linear system $AX = b$, $b \neq 0$ has no solution.

11. Which of the following statements is true?
- (A) Matrix multiplication is a vector-space operation on the set M of all $n \times n$ matrices.
- (B) Every vector space has at least two vectors.
- (C) Multiplication of any vector by the zero scalar always yields the zero vector.
- (D) The set Q of all rational numbers is a real vector space under the usual operations of addition and scalar multiplication.
- (E) None of the above.
12. Which of the following statements is true?
- (A) $\{(x_1, x_2, x_3) \mid x_1 + x_2 + x_3 = 1\}$ is a subspace of R^3 .
- (B) $S = \{(x, 1) \mid x \in R\}$ is a subspace of R^2 .
- (C) $S = \{(x_1, x_2, x_3) \mid x_1 = x_2\}$ is a subspace of R^3 .
- (D) If V and W are two subspace of R^n , then both $V \cap W$ and $V \cup W$ are subspaces of R^n .
- (E) None of the above.
13. Which of the following statements is not true?
- (A) The standard operations in R^n are vector addition and scalar multiplication.
- (B) The additive inverse of a given vector is not unique.
- (C) A vector space consists of four entities: a set of vectors, a set of scalars, and two operations.
- (D) A linearly independent spanning set S is called a basis of a vector space V .
- (E) The set of points on the line given by $x + y = 0$ is a subspace of R^2 .
14. The Wronskian for the given set of function $\{e^{-x}, xe^{-x}, (x+3)e^{-x}\}$ is
- (A) $2e^{-3x}$
- (B) e^{-3x}
- (C) $2e^{3x}$
- (D) e^{3x}
- (E) 0.
15. Let A and B be similar matrices, which one of the following statement is not true?
- (A) If A is singular, then so is B and A^{-1} and B^{-1} are similar.
- (B) A^T and B^T are similar.
- (C) There exists a matrix P such that $B^k = P^{-1}A^kP$.
- (D) If B is similar to C , then A is similar to C .
- (E) All of the above.
16. Let $T: R^3 \rightarrow R^2$ be a linear transformation such that $T(1) = 1 - x$, $T(x) = x + x^2$,

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$T(x^2) = 1 + 2x$ and $T(x^3) = 2 - x^2$, then $T(2 - 3x + x^2 - 2x^3) =$

- (A) $-2 + 3x + 2x^2$
- (B) $-2 + 3x - x^2$
- (C) $-2 - 3x - 2x^2$
- (D) $-1 + 3x - x^2$
- (E) $-1 - 3x - x^2$

17. Which statement is not true?

- (A) An $n \times n$ matrix P is orthogonal if and only if its column vectors form an orthonormal set.
- (B) Let A be an $n \times n$ symmetric matrix. If λ_1 and λ_2 are distinct eigenvalues of A , then their corresponding eigenvectors X_1 and X_2 are orthogonal.
- (C) Let A be an $n \times n$ matrix, then A is orthogonally diagonalizable and has real eigenvalues if and only if A is not symmetric.
- (D) If A and B are similar $n \times n$ matrices, then they have the same eigenvalues.
- (E) None of the above.

18. Let A be an $n \times n$ matrix, and let λ be an eigenvalue of A , then which one of the following statement is not true?

- (A) λ^k is an eigenvalue of A^k , $k=2,3,\dots$
- (B) If A is nonsingular, then $1/\lambda$ is an eigenvalue of A^{-1} .
- (C) If c is any scalar, then $\lambda + c$ is an eigenvalue of $A + cI$.
- (D) A and A^T have different eigenvalues.
- (E) None of the above.

19. Which of the following set of vectors in R^n is not orthogonal?

- (A) $\{(4, -1, 1), (-1, 0, 4), (-4, -17, -1)\}$
- (B) $\{(\frac{\sqrt{2}}{2}, 0, 0, \frac{\sqrt{2}}{2}), (0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0), (-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2})\}$
- (C) $\{(\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}), (-\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{6}), (\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3})\}$
- (D) $\{(\frac{\sqrt{2}}{3}, 0, -\frac{\sqrt{2}}{6}), (0, \frac{2\sqrt{5}}{5}, -\frac{\sqrt{5}}{5}), (\frac{\sqrt{5}}{5}, 0, \frac{1}{2})\}$
- (E) None of the above.

20. The matrix A' for $T: R^2 \rightarrow R^2, T(x_1, x_2) = (2x_1 - 2x_2, -x_1 + 3x_2)$, relative to the basis $B' = \{(1, 0), (1, 1)\}$ is

- (A) $A' = \begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix}$
- (B) $A' = \begin{bmatrix} 2 & 3 \\ -1 & -3 \end{bmatrix}$
- (C) $A' = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$
- (D) $A' = \begin{bmatrix} 2 & -2 \\ -2 & 1 \end{bmatrix}$
- (E) $A' = \begin{bmatrix} 2 & -2 \\ -1 & 2 \end{bmatrix}$

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二、填充題 (共 10 題; 每題 4 分)

21. 若 $\lim_{x \rightarrow 2} \frac{a\sqrt{x+7}+b}{x-2} = 2$, 則 $a+b =$ (21) .

22. $\int (e^{3x} + e^{2x})^{1/2} dx =$ (22) .

23. $\int_0^1 \int_x^1 e^{-y^2} dy dx =$ (23) .

24. $\lim_{t \rightarrow \infty} \frac{\int_0^{\sqrt{t}} (1-x)^2 dx}{\int_0^t \sqrt{x} dx} =$ (24) .

25. The interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$ is (25) .

26. If $f(x) = e^{-x^2}$, we have $f^{(2n)}(0) =$ (26) .

27. Given the set of data points as $\{(-2,2), (-1,1), (0,1), (1,3)\}$, then the least squares regression line for this set of data point is (27) .

28. The matrix that rotate counterclockwise by $\pi/3$ and then reflects about the line $y = -x$ is (28) .

29. For the vectors $u = (0,1,3,-6)$ and $v = (-1,1,2,2)$, the orthogonal projection of u onto v is (29) .

30. If $A = \begin{bmatrix} 5 & -6 \\ 3 & -4 \end{bmatrix}$, then $A^{10} =$ (30) .