明新學報 31 期 pp.137-145
Volume 31, Ming Hsin Journal, October 2005

Finding the Nondominated Solution Set of a Fuzzy Multiobjective Linear Program by Using Fuzzy Ranking Method

Miao-Ling Wang
Department of Industrial Engineering and Management, Ming Hsin University of Science and Technology, Hsinchu, Taiwan, R.O.C

Abstract

In this study, we focus on the fuzzy multiple objective decision making problem with fuzzy cost coefficients, right hand-sides and constraint matrix simultaneously. Following the analysis of our previous studies, by introducing the concept of an α-cut, the parameters will be transformed into interval-valued, but the convexity of the feasible region may not stand. Therefore, in this study, the left hand-sides and the right hand-sides of a constraint can be considered as a comparison of two fuzzy numbers. The methods of fuzzy ranking will be introduced to solve the problem in order to find the nondominated solution set. After defining the complete efficient solution set, a decision maker's preference is articulated based on his/her ranking order and the desired levels for the objectives if provided; otherwise, the principle of "more is better" in maximization problems is incorporated into the decision procedure. Theoretical evidences are provided with numerical illustrations.

Keywords：fuzzy number, fuzzy ranking, ε-constraint method, intra-parametric analysis, solution-mixes

應用模糊排序求解多目標線性規劃問題

王妙伶
明新科技大學工業工程與管理系

摘要

長久以來，在建立一個線性規劃模式時參數的給定往往十分複雜，除由決策者主觀認定外，不外乎利用歷史資料加以推算，然而即便是已給定之係數亦有可能由於時空之變化、外在環境的改變，參數必須隨之而改變，因此對不確定性之參數之研究是有其必要的。若參數爲一模糊數，延續以往在多目標線性規劃之研究成果，當成本係數爲模糊數可得一模糊解，同時當資源係數爲模糊數亦可得其模糊解。

本研究將發展一套模糊排序法。可完全區分出四個參數以下之同類型模糊數。進一步就限制矩陣亦加以模糊化，將形成模糊多目標決策(fuzzy multiple objective decision making)問題，如延續以往之研究方法，將模糊數以一切割數導入後形成區間係數之問題，然可行解區域之凸性(convex property)性質不存在，因此本研究將導入模糊排序(fuzzy ranking)之方式，將限制式之左右邊視爲兩組模糊數之大小比較，並進而將之去模糊化後求得非凌駕解(nondominated solution)。然排序的方法有許多，如何找到適合的方式亦爲本研究之重點，在比較不同方式後以較佳之排序方法來發展求解模式。

關鍵詞: 模糊排序、模糊數、參數間容忍度分析、解組合

1. Introduction

To estimate the exact values of a multiobjective linear programming is a problematic task. Normally, the coefficients are given by the decision maker, by the historical data or by the statistical inference. The stability is doubtful. Therefore, it is reasonable to construct a problem with imprecise coefficients. In our previous studies [22-25], we have developed theoretical results and solution procedures for a multiobjective linear program problem with the inexact coefficients. When the cost coefficients and the right-hand sides are fuzzy numbers individually and simultaneously, the solution procedures and decision procedures are proposed.

In this study, we focus on the fuzzy multiple objective decision making (FMODM) problem with fuzzy cost coefficients, right hand-sides and constraint matrix simultaneously as follows:

$$Max \quad \tilde{Z} = (\tilde{z}^1, \tilde{z}^2, ..., \tilde{z}^K)^t = (\tilde{c}^1\mathbf{x}, \tilde{c}^2\mathbf{x}, ..., \tilde{c}^K\mathbf{x})^t$$

$$s.t. \quad \tilde{\mathbf{A}}\mathbf{x} \le \tilde{\mathbf{b}}$$

$$\mathbf{x} \ge 0 \qquad (1)$$

where t means "transpose", $\tilde{\mathbf{A}} = [\tilde{a}_{ij}]$, for i =1,…,m, j =1,…,n is an $m \times n$ matrix with $\tilde{a}_{ij}$ being the fuzzy numbers of the constraint matrix ; $\tilde{\mathbf{b}}$ = [$\tilde{b}_i$], for i =1,…,m is the column vector with $\tilde{b}_i$ being the fuzzy numbers of the right-hand sides (RHSs) and $\tilde{\mathbf{c}}^k = [\tilde{c}_j^k]$, for k=1,…,K, j=1,…,n are the row vectors with $\tilde{c}_j^k$ being the fuzzy numbers of the cost coefficients respectively. In Section 2, we first focus on the existing method for FMODM. In Section 3, after reviewing the existing fuzzy ranking methods, by introducing a ranking function, we use the developed technique of intra-parametric programming to solve the question. By using a lexicographical decision procedure a desirable solution can be derived. A numerical example is illustrated in Section 4. Finally, discussion and conclusion are drawn in Section 5.

2. Literature Review

As regard to the imprecise coefficient problems, Oettli and Prager [15] have solved a linear system with interval-valued coefficients. Then, many investigations subsequently followed and were primarily concerned with the bounds of a solution set but not the exact solutions [1, 3, 15]. Until 1989, both theories and solution procedures for finding the exact bounds of a solution set have been obtained, yet the exact solution mixes still remain unknown. In other words, no decision can be made even if exact solution bounds are obtained. This is because that a solution which is chosen arbitrarily within the bounds is most likely an infeasible solution.

With regard to an optimization problem, Bitran [1] and Steuer [19] developed some algorithms to solve an MOLP problem of which the cost coefficients are interval-valued. By applying the Vector-Maximization Theory [20], an interval-valued MOLP can be transformed into a constant-valued problem, then a nondominated set can then be obtained [5, 6, 7, 20] by the found efficient extreme points. However, the algorithms are not easily implemented and the complete nondominated set can not be obtained.

Delgade *et al.* [4] have investigated a mathematical program with interval-valued cost coefficients from the viewpoint of fuzzy set theory. They assumed that a decision maker(DM) has the capability of assessing whether the values of the coefficients are near to the left, the right or the middle of the intervals. Then, a membership function can be defined in each interval to account for those options. Therefore, a fuzzy solution can be obtained.

Wang and Wang [22-25] had developed theoretical properties of an MOLP with fuzzy cost coefficients and right-hand sides simultaneously and individually, solution procedures and decision procedures are proposed to find the fuzzy nondominated solutions and make a most desirable solution from an efficient solution set.

Many researches have been conducted on the possibility theory. Ramik & Rimanek [16] had defined the inequality relation to transform fuzzy inequalities into a system of crisp inequalities. Lai and Hwang [9] had combined the fuzzy ranking concept in [21] with a developed strategy for imprecise objectives. After giving a minimal acceptable possibility β, a crisp MOLP problem cab be solved with different β, but the problem will be augmented into triple number of objectives and constrains. Negi [14] used the concept of exceedance possibility, after giving a lower cut value α of the exceedance possibilities of objectives and constrains, a crisp model with more than triple number of constraints can be solved. Luhandjula [13] had defined a satisfying solution by introducing the concepts of α-possible feasibility and β-possible efficiency. Different mixes of α and β will generate different solutions. Li and Lee [10] had transferred the fuzzy coefficients into parametric coefficients by giving an efficiency level α. Then by choosing different α value, one can obtain a set of different solutions. Slowinski [18] had summarized the existing methods and developed a solution procedure based on Hamming distance for comparison. Sakawa and Yano [17] then develop an interactive fuzzy satisfying method after giving an α - optimal level.

Before we proceed the analysis, let us introduce some concepts on fuzzy numbers as follows:

Definition 1: A convex and normalized fuzzy set defined on real numbers whose membership function is piecewise continuous is called a *fuzzy number* $\widetilde{M}=\{(x,\mu_{\widetilde{M}}(x))\,|\,x\in X,\mu_{\widetilde{M}}(x)\in[0,1]\}$.

Definition 2: For each $\alpha\in[0,1]$, an α-level set of a fuzzy number $\widetilde{M}$ is a crisp set whose membership values are greater than and equal to α denoted by $M_\alpha=\{x\,|\,x\in X,\mu_{\widetilde{M}}(x)\geq\alpha\}$.

Thus for each fuzzy number, there is at least a point whose membership value is 1 (normal); and the level set of each $\alpha\in[0,1]$ is a closed interval (convex).

Definition 3: A fuzzy number $\widetilde{M}$ is a trapezoidal fuzzy number if its membership function $f_{\widetilde{M}}$ is given by

$$f_{\widetilde{M}}(x)=\begin{cases}(x-m')/(m'-\underline{m}), & \underline{m}\leq x\leq m'\\ 1, & m'\leq x\leq m''\\ (\overline{m}-x)/(\overline{m}-m''), & m''\leq x\leq\overline{m}\\ 0, & otherwise\end{cases},\quad m',m'',\underline{m},\overline{m}\in R$$

, and can be denoted as $(m',m'',\underline{m},\overline{m})$.

3. An MOLP with the Trapezoidal Fuzzy Number Coefficients

In this study, without loss of generality, we shall focus a fuzzy MOLP with the trapezoidal fuzzy numbered coefficients. By using interval arithmetic, Model (1) will be transformed as:

$$\begin{aligned} Max\quad \widetilde{Z}&=(\widetilde{z}^1,\ldots,\widetilde{z}^K)^t\\ &=((\sum_{j=1}^n\underline{c}_j^1x_j,\sum_{j=1}^n c_j'^1x_j,\sum_{j=1}^n c_j''^1x_j,\sum_{j=1}^n\overline{c}_j^1x_j),\ldots,(\sum_{j=1}^n\underline{c}_j^Kx_j,\sum_{j=1}^n c_j'^Kx_j,\sum_{j=1}^n c_j''^Kx_j,\sum_{j=1}^n\overline{c}_j^Kx_j))^t \end{aligned}$$

$$s.t.\qquad s.t.\quad (\sum_{j=1}^n\underline{a}_{ij}x_j,\sum_{j=1}^n a_{ij}'x_j,\sum_{j=1}^n a_{ij}''x_j,\sum_{j=1}^n\overline{a}_{ij}x_j)\leq(\underline{b}_i,b_i',b_i'',\overline{b}_i),i=1,\ldots,m \tag{2}$$

$$x_j \geq 0, j = 1,\ldots,n$$

The main question of Model (2) to be answered consists in the left-hand and right-hand sides of comparison of the objectives and the constraints of fuzzy coefficients problems. There are two main categories of comparison functions and ranking functions. Comparison functions are defined by the relation of two fuzzy subsets, then map these two fuzzy subsets to a real number. Ranking functions map a fuzzy number to a real number, then these real numbers are compared. Many authors had tried to develop various ranking methods in order to generate a totally ordered set of fuzzy numbers. Bortolan and Degani [2] have drawn a review of some existing ranking method. In recent years, Yoon [28] converts a complex fuzzy number into probability density functions then the larger mean is the larger one. Fortemps and Roubens [8] proposed an area compensation procedure to compare fuzzy numbers. Lin [11] had summarized the taxonomy of the existing ranking methods and. the corresponding defects of them. The author had proposed a ranking function to develop a completely ordered of sigmoid fuzzy numbers[26, 27]. It had been proved that at most four steps of the proposed method generates a totally ordered set of any two fuzzy membership functions of the same type of the Gaussian, trapezoidal and triangle fuzzy numbers by comparing the mode, the right spread with respective to the mode, the total area below the membership and the upper-bound of 1-level set lexicographically.

However, in order to give the complete information, we introduce the method proposed by Liou and Wang [12] in this study. After computing the total integral value, a fuzzy number can be transformed into a function of α which is defined as an index of optimism α, $\alpha \in [0,1]$ to reflect the decision maker's optimistic attitude. A larger α means a higher α degree of optimism. That is, when α =0, the total integral is equal to the area left to the fuzzy number, which represents the most pessimistic viewpoint of decision maker and the fuzzy number is ranked into the smallest number as the midpoint of the left side of the fuzzy number, i.e. $\frac{\underline{m}+m'}{2}$. Conversely, when α =1, the total integral is equal to the total area which is left to and below the fuzzy number, which represents the most optimistic viewpoint of decision maker and the fuzzy number is ranked into the largest number as the midpoint of the right side of the fuzzy number, i.e. $\frac{\overline{m}+m''}{2}$. Therefore, we have the following problem:

$$Max \quad Z(\alpha)$$

$$= \left(\sum_{j=1}^{n}\left(\frac{c_j'^1 + \underline{c}_j^1}{2} + \frac{c_j''^1 + \overline{c}_j^1 - c_j'^1 - \underline{c}_j^1}{2}\alpha\right)x_j, \ldots, \sum_{j=1}^{n}\left(\frac{c_j'^K + \underline{c}_j^K}{2} + \frac{c_j''^K + \overline{c}_j^K - c_j'^K - \underline{c}_j^K}{2}\alpha\right)x_j\right)^t$$

$$s.t. \quad \sum_{j=1}^{n}\left(\frac{a_{ij}' + \underline{a}_{ij}}{2} + \frac{a_{ij}'' + \overline{a}_{ij} - a_{ij}' - \underline{a}_{ij}}{2}\alpha\right)x_j \leq \frac{b_i' + \underline{b}_i}{2} + \frac{b_i'' + \overline{b}_i - b_i' - \underline{b}_i}{2}\alpha, i = 1,\ldots,m \qquad (3)$$

$$x_j \geq 0, j = 1,\ldots,n, \alpha \in [0,1]$$

That is, we have a new crisp model with parametric cost coefficients, right-hand sides and functional constraints. In order to derive the complete information of the solution set, the following definitions are introduced:

Definition 4: A solution is said to be nondominated in Model (3) if at least one of the objective intervals is not dominated by the corresponding objective interval of other solutions.

Definition 5: A solution **x** is said to be an exact solution-mix of an interval-valued problem defined in (3) if some coefficients belong to their corresponding intervals such that **x** is a nondominated solution of such a problem.

Definition 6 [23]**:** An intra-parametric analysis of an MOLP is a parametric programming in which the tolerance regions found by different levels of parameters is defined for the same type of coefficients.

Different α will derive different nondominated solution set in Model (3). In order to obtain the complete solution set, intra-parametric analysis [23] provides a tool to find all critical regions of the possible optimal bases and their derived nondominated sets. As regard to the exact solution mixes, after defining the complete efficient solution set, a decision maker's preference is articulated based on his/her ranking order and the levels of desire for the objectives if provided; otherwise, the principle of "more is better" in maximization problems [24]. First, to articulate a decision maker's preference by presenting the best values which can be achieved by the respective objectives. Then, the order of importance of the objectives and if it is possible, the levels of desire on the objectives are requested from the DM. Second, a decision based on the ranked order can be obtained in such a manner that from the estimated sub-region of the parameters which satisfies the desired level of the most important objective, finding the one in the sub-region which can make the best value of the 2nd important objective is the desired value of the parameters and so forth. Consequently, the DM can obtain a satisfactory decision in an effective and flexible manner.

Thus, finding all nondominated solution involves finding all solutions from all possible combinations of coefficients. Briefly, this study involves (i) finding the nondominated set, (ii) identifying the exact solution-mixes.

4. Numerical Illustrations

In this section, let us consider an example below and illustrate the proposed method:

Example 1:

$$
\begin{aligned}
&Max \quad z^1 = (50,50,55,60)x_1 + (60,60,60,80)x_2 \\
&Maz \quad z^2 = (10,12,15,20)x_1 + (0,\frac{10}{3},\frac{10}{3},10)x_2 \\
&s.t. \quad (1,3,3,5)x_1 + (3,5,6,7)x_2 \le (100,140,140,160) \\
&\qquad (0,2,2,4)x_1 + (1,2,2,3)x_2 \le (60,90,90,90) \\
&\qquad x_1, x_2 \ge 0
\end{aligned} \tag{4}
$$

1º ***α-index transformation***

For each index of optimism α, we can obtain the following parametric problem (5):

$$
\begin{aligned}
&Max \quad z^1 = (50+7.5\alpha)x_1 + (60+10\alpha)x_2 \\
&Maz \quad z^2 = (11+6.5\alpha)x_1 + (\frac{5}{3}+5\alpha)x_2 \\
&s.t. \quad (4+2\alpha)x_1 + (2+2\alpha)x_2 \le 120+30\alpha \\
&\qquad (1+2\alpha)x_1 + (1.5+\alpha)x_2 \le 75+15\alpha \\
&\qquad x_1, x_2 \ge 0, \alpha \in [0,1]
\end{aligned} \tag{5}
$$

2º finding the nondominated set of P(0)

When the α level is given as 0, we have the following problem (6):

$$Max \quad z^1 = 50x_1 + 60x_2$$
$$Maz \quad z^2 = 11x_1 + \frac{5}{3}x_2$$
$$s.t. \quad 4x_1 + 2x_2 \le 120$$
$$x_1 + 1.5x_2 \le 75 \qquad (6)$$
$$x_1, x_2 \ge 0$$

Next, the procedure of the ε-constraint method are performed in the following step. First, the maximum of z^1 is $\bar{z}^1 = 3075$ which happens at $\mathbf{x}^* = (7.5,45)^t$,; that of z^2 is $\bar{z}^2 = 330$ at $\mathbf{x}^* = (30,0)^t$.

Consequently, the minimum of z^2 is 157.5 and we have problem (7) to be solved.

$$Max \quad z^1 = 50x_1 + 60x_2$$
$$s.t. \quad -11x_1 - \frac{5}{3}x_2 \le -157.5 - 172.5\varepsilon$$
$$4x_1 + 2x_2 \le 120 \qquad (7)$$
$$x_1 + 1.5x_2 \le 75$$
$$x_1, x_2 \ge 0, \varepsilon \in [0,1]$$

The nondominated solutions can be obtained as $(x_1^*, x_2^*) = (7.5 + 22.5\varepsilon, 45 - 45\varepsilon)^t, \varepsilon \in [0,1]$ and shown in the bold line of Figure 1.

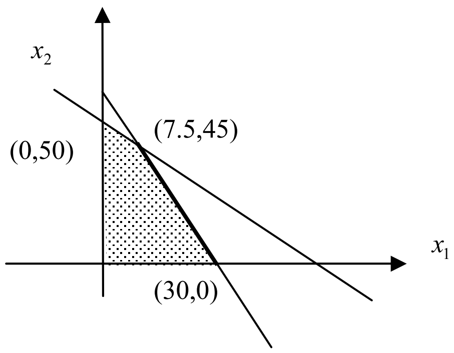

Figure 1 The Nondominated Set of P(0)

3° finding the nondominated set of P(1)

When the α level is given as 1, we have the following problem (8):

$$Max \quad z^1 = 57.5x_1 + 70x_2$$
$$Maz \quad z^2 = 17.5x_1 + \frac{20}{3}x_2$$
$$s.t. \quad 6x_1 + 4x_2 \le 150$$
$$3x_1 + 2.5x_2 \le 90 \qquad (8)$$
$$x_1, x_2 \ge 0$$

First, the maximum of z^1 is $\bar{z}^1 = 2520$ which happens at $\mathbf{x}^* = (0,36)^t$,; that of z^2 is $\bar{z}^2 = 437.5$ at $\mathbf{x}^* = (25,0)^t$ and the minimum of z^2 is 240. After performing the procedure of the ε-constraint method and we have problem (9) to be solved.

$$
\begin{aligned}
&Max \quad z^1 = 57.5x_1 + 70x_2 \\
&s.t. \quad -17.5x_1 - \frac{20}{3}x_2 \le -240 - 197.5\varepsilon \\
&\qquad 6x_1 + 4x_2 \le 150 \\
&\qquad 3x_1 + 2.5x_2 \le 90 \\
&\qquad x_1, x_2 \ge 0, \varepsilon \in [0,1]
\end{aligned} \tag{9}
$$

The nondominated solutions can be obtained as:

$(x_1^*, x_2^*) = \begin{cases} (\frac{395}{19}\varepsilon, 36 - \frac{474}{19}\varepsilon)^t, \varepsilon \in [0, \frac{19}{79}] \\ (-\frac{4}{3} + \frac{79}{3}\varepsilon, 39.5 - 39.5\varepsilon)^t, \varepsilon \in [\frac{19}{79}, 1] \end{cases}$ and shown in the bold line of Figure 2.

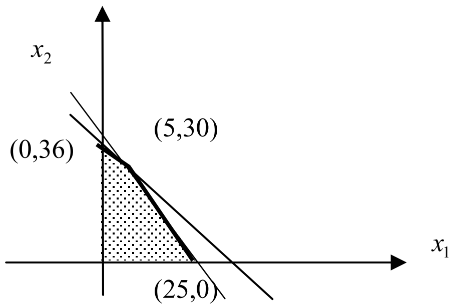

Figure 2 The Nondominated Set of P(1)

4º Finding the solution-mixes and decision analysis:

Since different α will generate different nondominated set, if the DM is an optimist, i.e. the index of optimism α is given as 1, for each solution-mix, the objective values are $2520 - \frac{20935}{38}\varepsilon$, $240 + \frac{22515}{114}\varepsilon$, if $\varepsilon \in [0, \frac{19}{79}]$ respectively and $\frac{8065}{3} - \frac{7505}{6}\varepsilon$, $240 + \frac{1185}{6}\varepsilon$, if $\varepsilon \in [\frac{19}{79}, 1]$ respectively. The largest interval of objective 1 is [1437.5, 2520], that of objective 2 is [240, 437.5]. Now, let us request a DM to rank the order of the objectives and if the DM considers that objective 1 is more important than objective 2 and requires that the desired level of objective 1 is 2000. Then, we have $\varepsilon = \frac{826}{1501}$, the exact solution-mix of $x^* = (\frac{750}{57}, \frac{675}{38})^t$, and this solution is the optimal solution which achieves the value of objective 2 being $348\frac{13}{19}$ under the requirement that the desired level of objective 1 is 2000.

5. Conclusions and Discussion

In this study, an MOLP with fuzzy cost coefficients, fuzzy constraint matrix and fuzzy RHSs is focused. The left hand-sides and the right hand-sides of a constraint can be considered as a comparison of two fuzzy numbers. After reviewing the existing fuzzy ranking methods, the index of optimism α, $\alpha \in [0,1]$ is introduced to reflect the decision maker's optimistic attitude to compute the total integral value, a fuzzy number can be transformed into a function of α . Therefore, we can use the developed technique of intra-parametric programming to solve the question. By using a lexicographical decision procedure a desirable solution can be obtained. Theoretical evidences are provided with numerical illustrations.

Further tasks of investigating a systematic solution procedure cooperating with more efficient ranking methods will be carried out in the near future.

Acknowledgement : The author acknowledge the financial support from the Ming Hsin University of Science and Technology, Taiwan, ROC, with the project number MUST-92-IEM-001.

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